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A Note On Zeroes Of Superpotentials In F-Theory

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We discuss the dependence of superpotential terms in 4D F-theory on moduli parameters. Two cases are studied: the dependence on world-filling 3-brane positions and the dependence on 2-form VEVs. In the first case there is a zero when the 3-brane hits the divisor responsible for the superpotential. In the second case, which has been extensively discussed by Witten in 3D M-theory, there is a zero for special values of 2-form VEVs when the M-theory divisor contains non-trivial 3-cycles. We give an alternative derivation of this fact for the special case of F-theory.

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1. Introduction

Super-potential terms in compactifications of M-theory to 3D on a Calabi-Yau 4-fold X arise from 5-branes wrapping non-trivial 6-cycles $D \subset X$ which satisfy [1]:

$$\sum (-)^p h^p(D) = 1. \tag{1.1}$$

If X is elliptically fibered over the 3-fold B one can take the F-theory limit of zero size fibers [2] and find a super-potential in the 4D, $\mathcal{N} = 1$ theory from a 3-brane that wraps a 4-cycle E inside the base B. It was shown in [3] that in 3D M-theory compactifications there are

$$n_{2-branes} = \frac{1}{24}\chi(X) \tag{1.2}$$

world-filling 2-branes which sit at a point on X (if χ is not divisible by 24, it was shown in [4] that one has to add 4-form field-strengths in M-theory which modify this number).

The super-potential W is a sum of terms corresponding to 5-branes wrapping divisors in X which satisfy (1.1) [1]. Each separate term is of the form

$$f(\cdots)e^{-V_D + i\phi_D} \tag{1.3}$$

where V_D is the volume of the divisor and

$$\phi_D = \int_D \widetilde{C}. \tag{1.4}$$

Here $\widetilde{C}$ is the 6-form dual of the 3-form of M-theory. The remaining pre-factor $f(\cdots)$ depends holomorphically on the moduli and should be found from a 1-loop computation – integrating all the massive fields on the world-volume of the 5-brane.

The moduli on which f could depend are thus:

a. The Kähler class of X which joins the 3D duals of C integrated on 2-cycles of X to form a complex scalar,
b. The complex structure of X,
c. The $h^{2,1} + h^{1,2}$ scalars which come from C,
d. The positions of the world-filling 2-branes.

In the F-theory limit, one takes the fibers to zero size. M-theory then "grows" an extra decompactified dimension, the KK modes of which are the almost massless states of 2-branes wrapped on the small fiber. In F-theory the superpotential is produced from 3-branes wrapping a (4D) divisor E in B whose pull-back to a (6D) divisor in X satisfies (1.1) [1]. The superpotential term is still given by an equation similar to (1.3):

$$f(\cdots)e^{-V_E + i\phi_E} \tag{1.5}$$

but V_E is now the volume of E and

$$\phi_E = \int_E B_4, \tag{1.6}$$

where B_4 is the self-dual 4-form of type-IIB. The world-filling 2-branes become world-filling 3-branes [3]. In the list above, positions of 2-branes become positions of 3-branes on B. The modes of C become the modes of the two RR and NSNS 2-forms $B^{(RR)}_{\mu\nu}, B^{(NS)}_{\mu\nu}$.

In this paper we will discuss the behavior of the super-potential as a function of the positions of the 3-branes and of the modes of the two-forms.

In the first case, we will find that f has a simple zero when the 3-brane hits the divisor E. The second case is a special case of [5]. In 3D M-theory compactifications, the prefactor f in [5] arises from the partition function of the 5-brane world-volume theory and the dependence on C arises from the "$dB \wedge C$" interaction on the 5-brane world-volume. In [5] the method for calculating the partition function for generic D's was derived. For generic D's, the difficulty is that there is no manifestly covariant Lagrangian for the chiral 2-form which lives on the 5-brane. In the F-theory limit, when D is elliptically fibered with fibers of zero size, the 5-brane world-volume theory reduces (roughly) to $\mathcal{N} = 4$ $U(1)$ Yang-Mills theory with a variable coupling constant and it is possible to calculate the partition function directly. We will do that in section (3) and verify that it is a section of the requisite bundle [5].

2. Dependence on 2-brane positions

We will start by analyzing the dependence on the 3-brane positions. For simplicity and more generality we will discuss the situation for 2-branes in M-theory.

Let us fix the positions of all 2-branes but one. The moduli space for that single 2-brane is a copy of X. We also keep all other irrelevant moduli frozen. At first sight, since X is compact and f has no obvious source of singularities (as we will soon see, a 2-brane that hits D doesn't produce a pole), we might think that f should be constant. That conclusion, however, would be wrong since f is a section of a *non-trivial* line-bundle $\mathcal{L}$ over the moduli space X. To see this we will examine $e^{i\phi_D}$. As noted in [1], ϕ_D is defined only up to a constant. As we change the position of the 2-brane to plot a small loop around D (the real codimension of D in X is 2) ϕ_D increases by 2π. To define ϕ_D in this patch of moduli space we deform D to D' which doesn't intersect the loop, pick M such that $\partial M = D - D'$ and set:

$$\phi_D = \int_M {}^*dC. \tag{2.1}$$

The 2-brane is a source for C and the loop intersects M once. Thus, after the 2-brane has finished the loop ϕ_D will pick up the flux from the 2-brane.

Geometrically, this means that $e^{-V_D+i\phi_D}$ is a section of the line bundle $[-D]$ associated with the divisor D. (This line bundle is defined as follows [6]: We take a neighborhood of D as one patch and the complement of D as the other patch. Let $g=0$ be a local defining equation for D, then $1/g$ is the transition function of the line-bundle when going from the complement of D to the neighborhood of D.) It follows that f is a section of $\mathcal{L}=[D]$. A holomorphic section of $[D]$ which has no poles must have a simple zero on an analytic manifold homotopic to D. If we assume that D is isolated ($h^3(D)=0$) for example, we find that f is zero everywhere on D.

We have seen that geometrically it is natural to expect a zero when a 2-brane hits a 5-brane, but where does this zero come from, physically?

2.1. What happens when a 5-brane hits a 2-brane?

We will argue that the zero comes from extra fermionic variables that live on the 5-brane. In general they are massive, but when a 2-brane sits on the 5-brane they become 2 massless anti-commuting variables that are *localized* at a point on the 5-brane. When one integrates them out one recovers the zero.

To argue the existence of these variables we take a (Euclidean) 5-brane spread in directions $5,6,7,8,9,10$ and a 2-brane in directions $0,1,2$. Compactifying the 10th direction we get a 4-brane and a 2-brane of type-IIA. Since they don't have any common 'time' we cannot use standard D-brane techniques, but we can 'grow' a time direction by compactifying and T-dualizing along the 4th direction, say.

We obtain a 5-brane and a 3-brane that intersect along the common direction 4. We have to find the *zero modes* along this common direction. We can now perform a Wick rotation and call that common direction 'time'. From the DD-strings we find two massless fermionic fields on the $0+1$ intersection. The fermions are charged under the difference of the $U(1)$-s that live on the two branes, but on the $0+1D$ intersection all that remains is A_0. So this gives a term

$$iA_0\psi_1\psi_2$$

There is also a Yukawa coupling

$$\Phi\psi_1\psi_2$$

where Φ is the separation in the 3rd direction. After T-duality back to the 4-brane and 2-brane, the Wilson loop A_0 becomes the separation in the 4th direction and $\Phi+iA_0$ combines to a complex parameter z that measures the displacement in directions $3,4$. The

zero-modes of $\psi(x_4)$ give two anti-commuting *variables* (with no coordinate dependence) that live at a *point* on the 5-brane and enter the Lagrangian via:

$$z\psi_1\psi_2.$$

Integration over ψ_1 and ψ_2 produces the pre-factor z which is the zero of the super-potential term.

2.2. Other examples of localized variables

We will briefly discuss two more cases where such localized variables can appear. The first is a world-filling 3-brane that hits a string world-sheet instanton. In M-theory, an instanton that corrects the Kähler metric can come from a membrane that wraps a 3-cycle. The corresponding instanton in F-theory would be a (p,q)-string whose boundary is on the 7-branes. The (p,q) type would change according to the $SL(2,\mathbb{Z})$ transformations of F-theory but the string is required to become elementary (i.e. $(1,0)$-type) on the 7-branes. The embedding map cannot be holomorphic (since it intersects the 7-branes over a real dimension one), but the area (in local string units) is required to be minimal. A presence of a world-filling 3-brane near the stringy instanton will add, as before, an extra localized variable. It is easy to analyze this by making an $SL(2,\mathbb{Z})$ transformation to make the string a D-string. As before, by T-duality this becomes a 2-brane intersecting a 4-brane at a point in space and everywhere in time. This intersection breaks supersymmetry but from the DD strings one finds again extra fermionic variables.

The other case is a world-filling 3-brane that approaches a type-IIB (-1)-brane. In this case we can T-dualize to a 3-brane parallel to a 7-brane. We find both fermionic and bosonic localized variables which cancel each other, so there is neither a zero nor a pole.

3. Dependence on 3-form moduli

In this section we will re-derive the result of [5] in a different way for the special case of an elliptically fibered divisor D with zero size fibers – the F-theory limit.

3.1. Review of Witten's results

We will start by reviewing some of the points from [5]. The space of VEVs of C is a hyper-torus of dimension $2h^{2,1}$. Let

$$C = \sum_{i=1}^{2h^{2,1}(X)} a_i L_i, \tag{3.1}$$

where

$$L_i, \qquad i = 1 \dots H^3(X) \tag{3.2}$$

is a basis of integral 3-forms on X.

Each a_i is periodic with period 2π. The complex structure on $T^{h^{2,1}}$ is given by the decomposition into $(2,1) \oplus (1,2)$ forms.

The $U(1)$-bundle on $T^{h^{2,1}}$ of which $e^{i\phi_D}$ is a section is determined from the kinetic term for ϕ_D in the 2+1D effective Lagrangian. One finds [5]:

$$\frac{1}{2}(d\phi_D + \frac{1}{4\pi}\sum_{ij} Q_{ij}(D) a_i da_j)^2, \tag{3.3}$$

where d is (2+1)-dimensional and the "charges" are given by

$$Q_{ij}(D) = \int_D L_i \wedge L_j. \tag{3.4}$$

This term arises from the $C\wedge dC\wedge dC$ interaction of M-theory.

From (3.3) we see that the kinetic term for ϕ_D has a connection term. The connection on the moduli space $T^{h^{2,1}}$ is given by

$$A_i = -\frac{1}{4\pi}\sum_j Q_{ij}(D) a_i. \tag{3.5}$$

Thus, $e^{i\phi_D}$ is a section of a $U(1)$-bundle $\mathcal{L}^{-1}$ with first Chern class:

$$c_1(\mathcal{L}^{-1}) = -\frac{1}{4\pi} Q_{ij}(D) da_i \wedge da_j. \tag{3.6}$$

Since D is an effective divisor, $\mathcal{L}$ is non-negative.

The pre-factor f in (1.3) will have zeroes on the moduli space, when there are two integral 3-forms on X whose pull-backs to $D \subset X$ have a non-trivial intersection $Q_{ij}(D)$. The zero should occur for special values of the integral of C on those 3-cycles.

The world-volume theory of the 5-brane is a 6D (twisted) tensor multiplet. The 3-form couples to $B^{(-)}$ via an interaction that looks locally like $\int C\wedge dB$ [7].

In [5] the partition function was calculated by combining the anti-self-dual $B^{(-)}_{\mu\nu}$ with a self-dual part so that it would be possible to write a covariant Lagrangian for them together. The partition function is then a sum of terms which are sections of different line bundles over the moduli space (all have the same Chern class (3.6) but they differ as complex line bundles). Separating from the partition function that piece which behaves as a section of the requisite line-bundle gives the required result.

3.2. F-theory limit

Let D be an elliptic fibration with base B. In the limit of zero-size fibers we have F-theory. The 5-brane becomes a 3-brane wrapped on B. The reduction of $B^{(-)}$ gives the $U(1)$ gauge field that lives on a 3-brane. To construct the partition function we need to divide B into patches $\cup_\alpha \mathcal{U}_\alpha$. The patches are connected by $SL(2,\mathbb{Z})$ transformations along the intersections $\mathcal{U}_{\alpha\beta}$. We will take the patches so that the intersections will be at the boundary of the patches, i.e. $\mathcal{U}_{\alpha\beta}$ will be 3-dimensional.

Locally on B we pick a basis $(\xi_1^{(\alpha)}, \xi_2^{(\alpha)})$ of integral 1-forms on the fiber such that they transform in the fundamental $SL(2,\mathbb{Z})$ representation. Their Hodge duals inside the fiber are

$$
{}^*\xi_1 = \frac{1}{\tau_2}(|\tau|^2\xi_2 + \tau_1\xi_1), \qquad {}^*\xi_2 = -\frac{1}{\tau_2}(\xi_1 + \tau_1\xi_2),
$$

where $\tau = \tau_1 + i\tau_2$ is the local modular parameter of the fiber.

The 3-form C becomes two real 2-forms, the NS-NS and the RR 2-forms of type-IIB:

$$
C|_{\mathcal{U}_\alpha} = B^{NS,(\alpha)}\wedge\xi_1^{(\alpha)} + B^{RR,(\alpha)}\wedge\xi_2^{(\alpha)} \tag{3.7}
$$

which combine into a single complex 2-form K:

$$
K^{(\alpha)} = \tau_1^{(\alpha)} B^{NS,(\alpha)} - i\tau_2^{(\alpha)}\, {}^*B^{NS,(\alpha)} - B^{RR,(\alpha)}. \tag{3.8}
$$

Over the patch $\mathcal{U}_\alpha$ the action is

$$
\begin{aligned}
I_\alpha =& \int_{\mathcal{U}_\alpha} \{ -\frac{1}{4g^2}(F - B^{NS})\wedge^*(F - B^{NS}) + \frac{i\theta}{32\pi^2}(F - B^{NS})\wedge(F - B^{NS}) \\
& - \frac{i}{2\pi} B^{RR}\wedge(F - B^{NS}) + fermions\} \\
=& \frac{1}{4\pi}\int_{\mathcal{U}_\alpha} \left\{ -\tau_2 F\wedge^*F + i\tau_1 F\wedge F + 2iK\wedge F + \Lambda(K,\bar{K}) + fermions \right\}, \\
\Lambda(K,\bar{K}) =& \frac{1}{4\pi}\int_{\mathcal{U}_\alpha} \left\{ -\tau_2 B^{NS}\wedge^*B^{NS} + i\tau_1 B^{NS}\wedge B^{NS} + 2iB^{RR}\wedge B^{NS} \right\},
\end{aligned} \tag{3.9}
$$

where $\tau = \frac{\pi i}{g^2} + \frac{\theta}{8\pi}$ (this is the normalization for $U(1)$ as opposed to $SU(2)$) is the complex parameter of the elliptic fiber.

When passing from one patch to the other the background variables are related by an $SL(2,\mathbb{Z})$ transformation:

$$
\begin{aligned}
\tau &\longrightarrow \frac{a\tau + b}{c\tau + d}, \\
B^{NS} &\longrightarrow aB^{NS} + bB^{RR}, \\
B^{RR} &\longrightarrow cB^{NS} + dB^{RR}.
\end{aligned} \tag{3.10}
$$

The gauge fields $A^{(\alpha)}$ and $A^{(\beta)}$ are related by an electric-magnetic duality transformation which is generically not an algebraic relation. One has to add a piece $I_{\alpha\beta}\{A^{(\alpha)}, A^{(\beta)}\}$ to the action. To find $I_{\alpha\beta}$ we can take $\mathcal{U}_{\alpha\beta}$ to be a plane at constant time $t = 0$. Then, $e^{iI_{\alpha\beta}}$ would be related to the operator that realizes electric-magnetic duality by [8]:

$$\Psi'\{A^{(\alpha)}\} = \int [DA^{(\beta)}] e^{iI_{\alpha\beta}\{A^{(\alpha)}, A^{(\beta)}\}} \Psi\{A^{(\beta)}\}. \tag{3.11}$$

Here Ψ and Ψ' are wave-functions before and after electric-magnetic duality.

One finds for $c \neq 0$:

$$I_{\alpha\beta}\{A, A'\} = \int_{\mathcal{U}_{\alpha\beta}} \left\{ \frac{a}{2c} A' \wedge dA' - \frac{d}{2c} A \wedge dA + \frac{1}{c} A \wedge dA' \right\}. \tag{3.12}$$

For $c = 0$ we have the T^b transformations:

$$e^{iI_{\alpha\beta}\{A, A'\}} = e^{\frac{ib}{2} \int_{\mathcal{U}_{\alpha\beta}} A \wedge dA} \delta\{A - A'\}. \tag{3.13}$$

Finally we have to take care of the 2D triple intersections $\mathcal{U}_{\alpha\beta\gamma}$. First we have to make sure that the gauge transformations between patches add up to integer multiples of 2π on $\mathcal{U}_{\alpha\beta\gamma}$ and second we have to add the two fermionic degrees of freedom which live on the intersection of a 7-brane with a 3-brane. To include the fermionic variables, we first choose the patches such that all the singular fibers correspond to a monodromy $T \in SL(2, \mathbb{Z})$, and then add:

$$I_{sing} = \int d^2x \, \psi D_z \psi, \tag{3.14}$$

where ψ is a chiral fermion that is charged with respect to the gauge field A of the patch. The gauge anomaly of I_{sing} is precisely what is needed to cancel the anomaly from (3.13) for the 3D cut corresponding to the T monodromy that has the 2D singular locus as a boundary.

Writing the partition function in full:

$$Z = Z_0 \int \prod_\alpha [\mathcal{D}A^{(\alpha)}][\mathcal{D}\psi] e^{-\sum_\alpha I_\alpha\{A^{(\alpha)}, K^{(\alpha)}\} + i\sum_{\alpha\beta} I_{\alpha\beta}\{A^{(\alpha)}, A^{(\beta)}\} + I_{sing}}. \tag{3.15}$$

If we switch back to Minkowskian metric we can find the classical equations of motion for this action:

$$d(\frac{1}{g^2} F^{(\alpha)} + \frac{\theta}{8\pi^2} {}^*F^{(\alpha)}) = 0, \tag{3.16}$$

and on the 3D intersections $\mathcal{U}_{\alpha\beta}$ we have

$$\begin{aligned}\frac{1}{(g^{(\alpha)})^2}\vec{E}^{(\alpha)}+\frac{\theta^{(\alpha)}}{8\pi^2}\vec{B}^{(\alpha)}&=a\left(\frac{1}{(g^{(\beta)})^2}\vec{E}^{(\beta)}+\frac{\theta^{(\beta)}}{8\pi^2}\vec{B}^{(\beta)}\right)+b\vec{B}^{(\beta)},\\ \vec{B}^{(\alpha)}&=c\left(\frac{1}{(g^{(\beta)})^2}\vec{E}^{(\beta)}+\frac{\theta^{(\beta)}}{8\pi^2}\vec{B}^{(\beta)}\right)+d\vec{B}^{(\beta)},\end{aligned}\tag{3.17}$$

where $\vec{E},\vec{B}$ are the electric and magnetic field on the 3D intersection. We can combine the $F^{(\alpha)}$ data to an anti-self-dual 3-form ω on the 6D manifold D. Writing locally

$$\omega^{(\alpha)}=F^{(\alpha)}\wedge\xi_1^{(\alpha)}+(\tau_1F^{(\alpha)}-i\tau_2{}^*F^{(\alpha)})\wedge\xi_2^{(\alpha)},\tag{3.18}$$

the $\omega^{(\alpha)}$'s join to form a closed anti-self-dual 3-form on D. For a generic metric, the solutions to equations (3.16) and (3.17) will not respect the integrality conditions on $\mathcal{U}_{\alpha\beta\gamma}$ and the anti-self-dual 3-form ω will not come out integral.

3.3. Transformation properties in $\mathcal{L}$

To prove that Z in (3.15) is a section of the line-bundle $\mathcal{L}$, we examine what happens as we change

$$C\longrightarrow C+2\pi L\tag{3.19}$$

where L is an integral 3-form on D.

We start with the simpler case of $D=B\times\mathbf{T}^2$. There is only one patch and the integral reads:

$$Z=e^{\Lambda(K,\bar{K})}Z',\qquad Z'=\int[\mathcal{D}A]e^{-\int\{\frac{1}{4g^2}F\wedge^*F+\frac{i\theta}{32\pi^2}F\wedge F+\frac{i}{2\pi}K\wedge F\}}.\tag{3.20}$$

Take χ to be an integral 2-form in $H^2(B,\mathbb{Z})$. Z' is invariant under

$$K\longrightarrow K+2\pi\chi.\tag{3.21}$$

The other period:

$$K\longrightarrow K+\frac{2\pi^2 i}{g^2}({}^*\chi)+\frac{\theta}{8}\chi\tag{3.22}$$

does not leave Z' invariant. In fact, (3.22) can be absorbed in a redefinition

$$F\longrightarrow F+2\pi\chi,\tag{3.23}$$

which is allowed since χ is integral. We see that up to terms that are independent of K (denoted C_0) we have

$$Z'(K + \frac{2\pi^2 i}{g^2}({}^*\chi) + \frac{\theta}{8}\chi) = C_0 e^{i\int_B K\wedge\chi} Z'(K). \tag{3.24}$$

From (3.24) it follows that Z' is a section of the line bundle $\mathcal{L}$ and so is Z.

Now we take the more general case of variable fibers. Assuming the patches are contractible, we expand

$$L|_{\mathcal{U}_\alpha} = dM_1^{(\alpha)}\wedge\xi_1^{(\alpha)} + dM_2^{(\alpha)}\wedge\xi_2^{(\alpha)}, \tag{3.25}$$

where $M_a^{(\alpha)}$ are local 1-forms on B. Substituting (3.25) and (3.18) in $\int C\wedge dB$ we find that (3.19) corresponds in the F-theory limit to

$$K^{(\alpha)} \longrightarrow K^{(\alpha)} + 2\pi\tau_1 dM_1^{(\alpha)} + 2\pi i\tau_2\, {}^*dM_1^{(\alpha)} + 2\pi dM_2^{(\alpha)}. \tag{3.26}$$

Note also that if we choose the patches so that all 7-branes are of $(1,0)$ type then M_1 vanishes on the 7-branes since the 1-cycle dual to ξ_1 vanishes on the 7-branes.

Now we change variables in (3.15):

$$A^{(\alpha)} \longrightarrow A^{(\alpha)} + 2\pi M_1^{(\alpha)}. \tag{3.27}$$

The integrality of L insures that the new $A^{(\alpha)}$ will satisfy the integrality condition on $\mathcal{U}_{\alpha\beta\gamma}$. Combining this with (3.26), the action changes by (dropping (α)):

$$\Delta I_\alpha = \int \{-iB^{NS}\wedge dM_2 + iF\wedge dM_2\}$$

To get rid of $F\wedge dM_2$ we integrate it on $\mathcal{U}_\alpha$ to get a contribution from the boundary only. This modifies $\mathcal{U}_{\alpha\beta}$ but together with the change due to (3.27) and the relation between $M_{1,2}^{(\alpha)}$ and $M_{1,2}^{(\beta)}$:

$$M_1^{(\beta)} = aM_1^{(\alpha)} + bM_2^{(\alpha)}, \qquad M_2^{(\beta)} = cM_1^{(\alpha)} + dM_2^{(\alpha)}. \tag{3.28}$$

(since M_1 vanishes on the 7-branes there is no change in I_{sing}), we find:

$$Z \to e^{-i\sum_\alpha \int_{\mathcal{U}_\alpha} B^{NS,\alpha}\wedge dM_2^{(\alpha)}} Z.$$

So, to sum up, if the 3-form C on D is given locally as

$$C|_{\mathcal{U}_\alpha} = K_1^{(\alpha)}\wedge\xi_1^{(\alpha)} + K_2^{(\alpha)}\wedge\xi_2^{(\alpha)}$$

and the integral 3-form L on D is given by

$$L|_{\mathcal{U}_\alpha} = L_1^{(\alpha)} \wedge \xi_1^{(\alpha)} + L_2^{(\alpha)} \wedge \xi_2^{(\alpha)}$$

The transition function of the bundle under $C \to C + 2\pi L$ is

$$e^{-i \sum_\alpha \int_{\mathcal{U}_\alpha} K_1^{(\alpha)} \wedge L_2^{(\alpha)}}$$

Changing to the real coordinates (3.1) on the moduli space:

$$C = \sum_i a_i L_i,$$

we see that under

$$a_i \longrightarrow a_i + 2\pi \delta_{ij}$$

we have the transition function (a sum over patches is implicit):

$$e^{i \sum_k a_k L_{2,k} \wedge L_{1,j}}. \tag{3.29}$$

We wish to show that (3.29) agrees with (3.6). To this end we must find a gauge transformation $\Lambda(a_1 \dots a_{2h^{2,1}})$ such that the connection

$$A_i = -\frac{1}{4\pi} \sum Q_{ij} a_j + \partial_i \Lambda$$

changes by (3.29):

$$A_k(a_1 \dots a_j + 2\pi \dots) - A_k(a_1 \dots a_j \dots) = \sum_\alpha \int L_{2,k} \wedge L_{1,j}.$$

Using

$$Q_{jk} = \sum_\alpha \int (L_{2,k} \wedge L_{1,j} - L_{1,k} \wedge L_{2,j})$$

We find

$$\Lambda = -\frac{1}{2} \sum a_j a_k \int (L_{1,k} \wedge L_{2,j} + L_{2,k} \wedge L_{1,j}). \tag{3.30}$$

4. Discussion

Studying the dependence of superpotentials on the moduli is important in order to determine whether supersymmetry is broken in a phase where a superpotential is generated. We have seen that super-potential terms from 5-branes wrapping divisors in M-theory develop a simple zero when a world-filling 2-brane sits on the divisor. The zero occurs because of the existence of extra light fermionic local variables on the 5-brane world-volume which appear when a 2-brane is close to the 5-brane. When there are n 2-branes sitting on the divisor one finds a zero of order n. In order to restore super-symmetry, one needs a zero of order two at least, so that the potential which is proportional to the square of the derivative of the superpotential will vanish.

In the second part of the paper we have verified that in the F-theory limit, the dependence of the super-potential terms on 3-form VEVs (which become RR and NS 2-forms in F-theory) agrees with the general results of [5]. It would be interesting to try to generalize the discussion of section (3) to calculate the partition function of two coincident 5-branes on an elliptically fibered 3-fold. This theory is related to the "tensionless string theories", and (3.12) should somehow be generalized to a nonabelian S-duality operator.

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Note added

As this work was completed, a paper [9] appeared which also discusses the extra localized variables at the intersection of a Euclidean and a world-filling 3-brane. In [9] the variables were bosonic, but this is probably related to the fact that the setting there was different and the world-filling 3-brane was in a different phase. The situation that was discussed in the present paper corresponds, in the context of [9], to giving the quarks masses. In this phase the superpotential has a zero when a quark mass vanishes. The superpotential is then proportional to a fractional power of the quark mass because the Euclidean 3-brane sits on $N+1$ 7-branes. In M-theory this corresponds to a 2-brane on an A_N singularity which means that the transverse coordinate is the Nth root of the mass.

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